Robust Normative Comparisons of Socially Risky Situations*

Nicolas Gravel[†]and Benoît Tarroux[‡]

May 26th 2014

Abstract

In this paper, we theoretically characterize robust empirically implementable *normative* criteria for evaluating *socially risky situations*. Socially risky situations are modeled as distributions, among individuals, of lotteries on a finite set of state-contingent pecuniary consequences. Individuals are assumed to have selfish Von Neumann-Morgenstern preferences for these socially risky situations. We provide *empirically implementable* criteria that coincide with the unanimity, over a reasonably large class of such individual preferences, of anonymous and Pareto-inclusive Von Neuman Morgenstern social rankings of risks. The implementable criteria can be interpreted as *sequential expected poverty dominance*. An illustration of the usefulness of the criteria for comparing the exposure to unemployment risk of different segments of the French and US workforce is also provided.

Keywords: Risk, Dominance, *ex ante* Social Welfare, State-Dependent expected utility

Expected Utility, Poverty, Unemployment.

JEL classification numbers: C81, D3, D63, D81, I32, J63, J64

1 Introduction

The exposure to *risks* that societies provide to their members is a clearly important ingredient for normative evaluation. For instance countries like the US or the UK are commonly depicted as having "flexible" labour markets in which most of the workforce faces a small probability of involuntary unemployment that, when it happens, receives little compensation and where the wages of those employed are high. Other countries, like France, are to the contrary portrayed as having "rigid" labour markets in which a significant fraction of the workforce is protected against the risk of involuntary unemployment even though it enjoys moderate wages, while the remaining part of the workforce

*This work has immensely benefited from the comments made by Conchita d'Ambrosio, Marc Fleurbaey, Patrick Moyes, Alain Trannoy and two anonymous referees. The usual disclaimer applies.

†Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS, Centre de la Vieille Charité, 2, rue de la Charité, 13 002 Marseille, France, nicolas.gravel@univ-amu.fr.

‡Université de Rennes 1, CREM & IDEP, 7 place Hoche, 35 065 Rennes Cedex, France. benoit.tarroux@univ-rennes1.fr.

is exposed to a high probability of unemployment which, if it arises, is the object of significant pecuniary compensation. A natural question to ask from a normative point of view is: what form of organization of the labour market is better? Analogously, one may be interested in comparing different countries - or the same country at different points in time - on the basis of their distributions of income and exposure to crime, to risks of health (see e.g. Gravel, Moyes, and Tarroux (2009) or Gravel and Mukhopadhyay (2010)) or in evaluating various programs of prevention against health or environmental risks (e.g. Viscusi (2007)).

In this paper, we theoretically justify and empirically implement dominance criteria that compare allocations of individuals' exposure to risks in an ethically robust fashion. The risks handled by our approach can be described, from the view point of the individual who faces them, by a finite list of probabilities of occurrence of *individual states of nature* (e.g. being unemployed or employed, gravely ill, mildly ill or perfectly healthy, etc.) and of pecuniary consequences contingent on those states. The risks can also have non-pecuniary consequences in the sense that the subjective evaluation of a given amount of money may depend upon the state of nature in which it is received. When risks have non-pecuniary consequences in this sense, the individual states of nature are assumed to be unambiguously ordered from the worst (e.g. being gravely ill) to the best (e.g. being in perfect health).

At first sight, normative comparisons of distributions of individual exposure to risk can be seen as particular instances of general *multi-dimensional* normative evaluation (see e.g. Atkinson and Bourguignon (1982), Kolm (1977)). After all the probability of being unemployed, falling ill, being the victim of a crime, etc. can be viewed as individual *attributes* whose distributions, along with that of income, can be evaluated by general multi-dimensional dominance tools of the Atkinson and Bourguignon (1982) variety (see e.g. Gravel, Moyes, and Tarroux (2009) or Gravel and Mukhopadhyay (2010)). Yet the *particular structure* imposed by the fact that the attributes are individual probabilities of falling in alternative states turns out, when interpreted through standard decision theory, to affect significantly the nature of the evaluative exercise. As will be seen indeed, the criteria that we obtain are quite different from the abstract first or second order *multi-dimensional* stochastic dominance criteria *à la* Atkinson and Bourguignon (1982). However, and as discussed below, they bear a formal similarity with the *two-dimensional* dominance criteria developed by Atkinson and Bourguignon (1987) and Jenkins and Lambert (1993) for ranking distributions of income between households differing by their need.

We assume that individuals have (possibly state-dependent) *Von Neumann-Morgenstern* (VNM) preferences over these risks. Acknowledging that a distribution of these individual risks can be seen as a *socially risky situation* (using Fleurbaey (2010) terminology), we obtain empirically implementable criteria for comparing these socially risky situations that *coincide* with the unanimity, taken over a reasonably large class of such individual

VNM preferences, of all *Pareto consistent* and *anonymous VNM* social rankings. As a result of a version of Harsanyi (1955)'s aggregation theorem due to Weymark (1991), it happens that any such ranking of socially risky situations can be numerically represented as the unweighted sum of the individuals' expected utility functions. If the social ranking is required to satisfy an anonymity requirement, and individual preferences are selfish, then these individuals' expected utility functions are all identical. Within this framework, we characterize *two* implementable criteria that correspond to different sets of assumptions made on these individual expected utility functions.

We first consider the class of individual VNM preferences that are represented by an expected utility function that exhibits a *positive* marginal utility of income that is *decreasing* with respect to the individual states (ordered from the worst to the best). We show that the empirically implementable criterion that coincides with the unanimity, over this class of individual preferences, of all Pareto inclusive, anonymous and VNM social preferences is what we call "*Sequential Expected Headcount Poverty*" (SEHP) dominance. This criterion ranks a socially risky situation A weakly above a socially risky situation B if, for any poverty line and any state, the *expected* number of individuals who are *both* below the line *and* in a worse state is no greater in A than in B. The sequential aspect of the criterion arises from the fact that in order to check for dominance, one looks first at expected number of poor in the worst state and, in a second step, at the total expected number of poor in the two worst states, and so on, in a sequential fashion. To that extent, this criterion may be viewed as giving a *priority to poverty* that is decreasing with the states. This reflects of course the assumption that the marginal utility of income is decreasing with respect to states, an assumption that may be at odds with empirical evidence (see e.g. Viscusi and Evans (1990)).

The second, more restricted, family of VNM preferences is made of those preferences that satisfy, in addition, the property of *risk aversion* (the marginal utility of income is decreasing with income in every state) and the (somewhat more contentious) condition that the decrease in the marginal utility of income with respect to income be itself decreasing with respect to the states. This later condition is implied by, but is not equivalent to, the requirement that the degree of absolute risk aversion, as measured by the Arrow-Pratt coefficient, be *decreasing* with the states. We show that the implementable criterion that coincides with the unanimity, taken over all individual VNM preferences in this class, of all rankings produced by a Pareto-inclusive and anonymous VNM social preference is what we call "*Sequential Expected Poverty Gap*" (SEPG) dominance. This criterion works just like the SEHP one, but with poverty gap, rather than headcount poverty, used as the poverty measure.

We also illustrate the usefulness of our criteria for comparing the distributions of risks of unemployment in France and in the US. We specifically show that our criteria do not enable one to rank US and France in terms of their allocation of unemployment risks.

The empirical illustration reveals also that, in France, male adults are better protected against the risk of unemployment than female ones while no such dominance of males over females is observed in the US. This suggests therefore that the male-female gap in protection against unemployment risks is higher in France than in the US. The analysis also reveals that young segments of the workforce have worse exposure to unemployment risks than older ones, but that this advantage of the old over the young is somewhat lower in the US than in France.

The plan of the remaining of the paper is as follows. The next section introduces the normative and empirically implementable criteria and establishes the formal equivalence between them. The third section applies the criteria to the US-France comparisons of the distribution of risks of involuntary unemployment. The fourth section concludes.

2 Theory

2.1 Normative criteria

We consider societies made of a *given* number, n say, of individuals[1], indexed by i, with $i \in N = \{1, ..., n\}$. Societies expose their individual members to *risks* of falling into a finite number, l say, of mutually exclusive *individual states* indexed by j and taken from some set $\Omega = \{1, ..., l\}$. We admit the possibility that individuals attach *intrinsic value* to the state in which they fall (as they may, for instance, prefer receiving a given amount of income while being healthy than while being ill). We do this by assuming that these individual states are ordered from the worst (state 1) to the best (state l). In addition to their intrinsic appeal, individual states are also valued for the pecuniary consequences (incomes) that the individual may get contingent upon them. In order to keep the formalism simple, we assume that the set $\mathbb{I}$ of all conceivable income levels is finite and can be written as $\mathbb{I} = \{0, ..., m\}$ for some (possibly large) integer m. Income is therefore assumed to be available in non-negative integer quantities (say in cents). Like Fleurbaey (2010), we call *socially risky situation* a specific pattern of individuals' exposures to risks. Formally, and somewhat differently than Fleurbaey (2010), we model a socially risky situation as a probability distribution - or *lottery* $-$ p on the set $\mathbb{X} = (\Omega \times \mathbb{I})^n$ of all logically conceivable n-tuples of individual state-income pairs, one such pair for every individual.[2] A typical element x of $\mathbb{X}$ writes:

$$x = (j_1^x, y_1^x, ..., j_n^x, y_n^x) \tag{1}$$

[1] The generalization to societies involving different numbers of individuals is immediate.

[2] In Fleurbaey (2010) socially risky situations are described as probability distributions over profiles of VNM utility levels. The restriction of the analysis to a finite set $\mathbb{X}$ is a pedagogical simplification that enables us to use the version of Harsanyi social aggregation theorem provided by Weymark (1991) as a theoretical justification to our criteria.

where, for $i = 1, ..., n$, $j_i^x \in \Omega$ denotes the *individual* state in which individual i falls in the *social* state x and $y_i^x \in \mathbb{I}$ denotes i's income in that social state. In this finite framework, a lottery p is just an element of the $(lm)^n - 1$ dimensional simplex, the xth component of which, denoted p_x, being interpreted as the probability that the society falls into x under the socially risky situation p. We abuse notation and, for any $x \in \mathbb{X}$, we denote simply by x the "non-risky" socially risky situation that assigns a probability one to social state x and a probability 0 to any other social state. We denote by $\mathbb{L}$ the set of all lotteries on $\mathbb{X}$.

Every individual $i \in N$ is assumed to have a selfish VNM preference ordering[3] $\succsim_i$ on $\mathbb{L}$, with asymmetric and symmetric factors $\succ_i$ and $\sim_i$ respectively. This means that there exists a function $U_i : (\Omega \times \mathbb{I}) \to \mathbb{R}$ such that, for every socially risky situation p and q in $\mathbb{L}$, one has:

$$p \succsim_i q \Leftrightarrow \sum_{x \in \mathbb{X}} p_x U_i(j_i^x, y_i^x) \geq \sum_{x \in \mathbb{X}} q_x U_i(j_i^x, y_i^x). \tag{2}$$

As we shall be concerned by anonymous social preference, we shall in fact assume that these functions U_i are the same for all individuals. Socially risky situations in $\mathbb{L}$ are normatively evaluated by a *social ordering* $\succsim$ that can be defined by:

$$p \succsim q \Leftrightarrow \sum_{x \in \mathbb{X}} p_x \sum_{i \in N} U(j_i^x, y_i^x) \geq \sum_{x \in \mathbb{X}} q_x \sum_{i \in N} U(j_i^x, y_i^x) \tag{3}$$

for some (common to all individuals) function $U : \Omega \times \mathbb{I} \to \mathbb{R}$. There are many ways by which one could justify a ranking of socially risky situations having such a utilitarian-like form. The one that we favour appeals to the well-known Harsanyi (1955) aggregation theorem. It is indeed not difficult to establish that any VNM ranking of socially risky situations that respect - in the Pareto sense - the individual selfish VNM preferences and that is anonymous will be represented as per (3) for some utility function U whose expectation represented every individual VNM preferences.

Each criterion proposed in this paper will be shown to rank lotteries in $\mathbb{L}$ in a way that coincides with the *unanimity* of all social orderings that can be written as per (3) for some individual utility function U taken from some (reasonably large) class. We call "normative dominance" any such criterion that is defined as follows.

Definition 1 (Normative dominance) *Socially risky situation p normatively dominates socially risky situation q for a class $\mathbb{U}$ of functions $U : \Omega \times \mathbb{I} \to \mathbb{R}$, denoted $p \succsim_{\mathbb{U}} q$, if inequality (3) holds for all functions U in the class.*

In the current paper, we apply this definition of normative dominance to two specific classes of state-dependent preferences. In order to define formally these classes in the dis-

[3] An ordering is a reflexive, complete and transitive binary relation.

crete setting considered herein, we first introduce the following properties of the expected utility representation of such state-dependent VNM preferences.

P1 For every state $j = 1, ..., l-1$, one has:
(i) $U(j,y) \leq U(j+1,y)$ for every income $y \in \{1,...,m\}$ and,
(ii) $0 < U(j+1,y+1) - U(j+1,y) \leq U(j,y+1) - U(j,y)$ for every income $y \in \{1,...,m-1\}$.

P2 For every state $j = 1, ..., l-1$ and income $y \in \{1,...,m-2\}$, one has:
$0 \geq U(j+1,y+2) - 2U(j+1,y+1) + U(j+1,y) \geq U(j,y+2) - 2U(j,y+1) + U(j,y)$

We denote by $\mathbb{U}_1$ the class of all state-dependent utility functions satisfying property **P1** and by $\mathbb{U}_2$ the (more restricted) class of such utility functions who satisfy both **P1** and **P2**.

An individual with VNM preferences whose expected utility representation uses a function in $\mathbb{U}_1$ prefers being for sure in a better state than in a worse one (given income), prefers receiving more income to less (given the state), and values more an additional unit of income enjoyed for sure in a bad state than in a good one. Preferences that are represented by an expectation of a utility function in $\mathbb{U}_2$ exhibit, in addition, risk aversion (the marginal utility of income decreases with respect to income in any state) as well as the requirement that the *decrease* in marginal utility of income be itself *decreasing* with respect to the individual state.

For a strictly risk averse preference[4] satisfying **P1**, it can be checked easily that the latter property is *implied by* the requirement that absolute risk aversion - as measured by the (discrete) Arrow-Pratt coefficient[5] - is decreasing with the state. On the other hand, assumption **P2** does not imply that risk aversion is decreasing across states. For instance, if $l = 2$, the function $U : \{1,2\} \times \mathbb{I}$ defined by:

$$U(j,y) = 1 - (3-j)e^{-3^{j-1}y} \tag{4}$$

satisfies **P1** and **P2** (the (continuous) graphs of $U(1,y) = 1 - 2e^{-y}$ and $U(2,y) = 1 - e^{-3y}$ are depicted on figure 1).

[4] A VNM preference is strictly risk averse if the function U whose expectation numerically represents it satisfies, for every $j \in \Omega$ and $y \in \{1,...,m-2\}$, $U(j,y+2) - 2U(j,y+1) + U(j,y) < 0$.

[5] For the individual state j and the income level y, the discrete Arrow-Pratt coefficient is the number $a(j,y)$ defined by:

$$a(j,y) = \frac{-[U(j,y+2) - 2U(j,y+1) + U(j,y)]}{U(j,y+1) - U(j,y)}$$

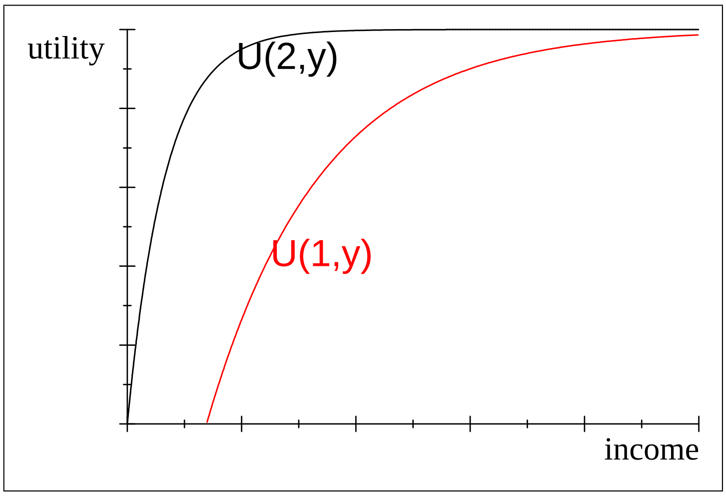

Figure 1

However:

$$
\begin{aligned}
a(1,y) &= \frac{-[-2e^{-(y+2)}+4e^{-(y+1)}-2e^{-y})]}{(-2e^{-(y+1)}+2e^{-y})} \\
&= \frac{e^{-2}-2e^{-1}+1}{-e^{-1}+1} \approx 0,63212 \\
&< 0.95021 \approx \frac{e^{-6}-2e^{-3}+1}{-e^{-3}+1} \\
&= \frac{-[-e^{-3(y+2)}+2e^{-3(y+1)}-e^{-3y}]}{-e^{-3(y+1)}+e^{-3y}} \\
&= a(2,y)
\end{aligned}
$$

so that the Arrow-Pratt coefficient of risk aversion of an individual with VNM preference represented by (4) is increasing with the state.

While the requirement that VNM preferences be increasing with respect to the state or the income for sure or be risk averse are standard and somewhat plausible, we recognize that the two other properties may not be. Consider first the property that the marginal utility of income be decreasing with respect to the state. When interpreted in a health context, it implies that an individual who could purchase health insurance at an actuarially fair premium would choose to over insure herself. Indeed, consider a two-state world ($\Omega = \{1,2\}$) and a household endowed with an income of y exposed to a probability π of having a health problem that can be insured at an actuarially fair premium of $r = \frac{\pi}{1-\pi}$. Assuming that the health problem, if it arises, generates a financial loss of L, one can write the optimal insurance purchase decision of that individual as:

$$\max_{I \geq 0} (1-\pi)U(2, w - \frac{\pi I}{1-\pi}) + \pi U(1, w - L + I)$$

Because any purchase of a strictly positive amount I^* of insurance will satisfy the first order condition of this program[6], one would have:

$$\partial U(2, w - \frac{\pi I^*}{1-\pi})/\partial y = \partial U(1, w - L + I^*)/\partial y$$

For this equality to hold given the assumed property of U one must have $w - L + I^* \geq w - \frac{\pi I^*}{1-\pi}$ and, therefore $I^* \geq (1-\pi)L$. Hence an individual with VNM preferences represented by the expectation of a utility function satisfying **P1** who has access to actuarially fair health insurance would choose to purchase *more* insurance than the expected financial loss incurred in case of a health problem. There seems to be some empirical evidence (see e.g. Viscusi and Evans (1990)) that such a behavior is not plausible. We are not aware of empirical evidence in favour of (or against) the property of decreasing risk aversion with respect to the state or of its implication in terms of **P2.**

We finally emphasize that, while the implementable criteria proposed in the next section of this paper are based on these properties, the approach is flexible. Other implementable criteria could possibly be produced by considering alternative sets of assumptions on VNM preferences.

2.2 The criteria and their characterization

For a socially risky situation $p \in \mathbb{L}$, we let $p(i, j, y)$ be the probability that individual i ends up in individual state j with income y in that socially risky situation. This probability is defined by:

$$p(i, j, y) = \sum_{\{x \in \mathbb{X} : j_i^x = j,\ y_i^x = y\}} p_x \tag{5}$$

With this notation, we *first* introduce the *Sequential Expected Headcount Poverty* (SEHP) dominance criterion.

Definition 2 (Sequential Expected Headcount Poverty dominance) *For p and $q \in \mathbb{L}$, we say that p SEHP dominates q, denoted $p \succsim_{SEHP} q$ if, for every poverty line $t \in \mathbb{I}$ and every state $k \in \Omega$, one has:*

$$\sum_{i \in N} \sum_{j \leq k} \sum_{y \leq t} p(i, j, y) \leq \sum_{i \in N} \sum_{j \leq k} \sum_{y \leq t} q(i, j, y) \tag{6}$$

In words, socially risky situation p dominates socially risky situation q for the SEHP criterion if, for every state k and monetary poverty line t, the *expected numbers* of individuals who are *both* in a weakly worse state *and* poor is no greater in p than in q. As the SEHP criterion requires inequality (6) to hold for every poverty line, it implies,

[6]We are cheating a bit here with respect to the discrete setting in which the rest of the analysis is conducted by considering a differentiable framework.

by choosing a large enough poverty line, that, for every state k, the expected number of individuals in states weakly worse than k be no greater in the dominating situation than in the dominated one. In the same spirit, since the SEHP criterion requires inequality (6) to holds for $k = l$, it implies the expected number of poor irrespective of the state to be no greater in the dominating situation than in the dominated one. Notice that requiring the *expected* number of poor irrespective of the state to be lower in the dominating situation is *not* equivalent to requiring the same relationship to hold for the *total* number of poor irrespective of the state. The only instance where the two requirements could coincide would be a socially risky situation in which the income received by an individual is the same in all states.

The *second* implementable criterion is the analogue of SEHP dominance, but with *poverty gap*, rather than headcount poverty, used as a measure of poverty. We call it, for this reason, the *Sequential Expected Poverty Gap* (SEPG) criterion. In order to define this criterion, we denote by $P(t, y)$ the poverty gap of income y for the poverty line t defined by:

$$P(t, y) = \max[t - y, 0] \tag{7}$$

This poverty gap is, as usual, interpreted to be the minimal amount of income required to get a person with income of y out of poverty when the poverty line is t. We accordingly define the SEPG criterion as follows.

Definition 3 (Sequential Expected Poverty Gap dominance) *For p and $q \in \mathbb{L}$, we say that p SEPG dominates q, denoted $p \succsim_{SEPG} q$ if, for every poverty line $t \in \mathbb{I}$ and every state $k \in \Omega$, one has:*

$$\sum_{i \in N} \sum_{j \leq k} \sum_{y \in \mathbb{I}} p(i, j, y) P(t, y) \leq \sum_{i \in N} \sum_{j \leq k} \sum_{y \in \mathbb{I}} q(i, j, y) P(t, y) \tag{8}$$

and, for every state k, it is the case that:

$$\sum_{i \in N} \sum_{j \leq k} \sum_{y \in \mathbb{I}} p(i, j, y) \leq \sum_{i \in N} \sum_{j \leq k} \sum_{y \in \mathbb{I}} q(i, j, y) \tag{9}$$

In words, socially risky situation p dominates socially risky situation q for the SEPG criterion if, for every poverty line and every state, the *expected amount of money* required to eliminate poverty *in all weakly worse states* is lower in p than in q (condition (8)) and if the expected number of individuals who are in a weakly worse state is also weakly smaller in p than in q (condition (9)). It should be noticed that requirement (9) must be *added* to the definition of the criterion rather than being deduced, as was the case for the SEHP criterion, from the single inequality (8). As can be checked from the proof of theorem 2 in appendix A, the additional requirement (9) can be dispensed with if one assumes the existence of a sufficiently high income level for which an individual is indifferent between states. We must admit that we find this assumption rather implausible. After all, even arbitrarily rich individuals experience utility loss when they are raped or severely injured.

We notice that the SEHP and SEPG criteria can be viewed as dominance criteria made on specific members of the class of "vulnerability to poverty" measures discussed in Dutta, Foster, and Mishra (2011). This is at least true for the SEPG criterion (as the SEHP criterion does not ride on a measure of "vulnerability to poverty" that belongs to the class characterized in Dutta, Foster, and Mishra (2011)).

We notice also that the SEPG dominance criterion is, from a formal point of view, similar to the one proposed by Bazen and Moyes (2003) for comparing distributions of incomes between households with differing needs (or abilities). In the need approach, one compares distributions, between n households (individuals) of pairs (j_i, y_i) (for $i = 1, ..., n$) where j_i is an (ordinal) index of i's ability and y_i is a (cardinally meaningful) measure of i's income. Ability levels can only take up finitely many values and look therefore very much like the individuals' "states" of the current approach.[7] From an abstract view point, one may indeed view the fact, for a given individual, of belonging to a given "need category" to be the *ex post* result of a socially risky situation. For any **d**istribution d of ability and income levels, one can denote by $f^d(j, y)$ the (possibly null) **f**raction of the population who have both ability j and income y in that distribution. Bazen and Moyes (2003) (see their proposition 4.1) have proposed, as a variant of Jenkins and Lambert (1993) generalization of Atkinson and Bourguignon (1987) sequential Lorenz criterion, to rank a distribution d above a distribution d' if and only if it is the case that, for every ability level $k \in \{1, ..., l\}$:

$$\sum_{j=1}^{k} \sum_{y \in \mathbb{I}} f^d(j, y) P(t, y) \leq \sum_{j=1}^{k} \sum_{y \in \mathbb{I}} f^{d'}(j, y) P(t, y) \tag{10}$$

for every poverty line t and:

$$\sum_{j=1}^{k} \sum_{y \in \mathbb{I}} f^d(j, y) \leq \sum_{j=1}^{k} \sum_{y \in \mathbb{I}} f^{d\prime}(j, y) \tag{11}$$

Inequalities (10) and (11) are clearly obtained from inequalities (8) and (9) that define SEPG dominance by simply setting:

$$f^d(j, y) = \sum_{i \in N} p(i, j, y)/n$$

[7] However, the need literature typically assumes that incomes are available in any (real) quantity whatsoever. This difference with the current setting is not essential.

and:

$$f^{d'}(j,y) = \sum_{i \in N} q(i,j,y)/n$$

Hence, the Bazen-Moyes or Jenkins-Lambert criteria can be said to be implied by the SEPG criterion if the observed frequency $f^d(j,y)$ of the population in state j and in income y turns out to coincide precisely with its expectation $\sum_{i \in N} q(i,j,y)/n$. Yet there is no reason for such a precise coincidence to hold. The "need approach" is, in effect, an *ex post* approach that compares distributions of outcomes *after* the uncertainty has been resolved and the individuals have been assigned to their final "need category-income" pair. The current analysis, on the other hand, looks at things from an *ex ante* perspective, and focuses on the distributions of risks *before* the uncertainty has been resolved.

In order to make that point clear, let us consider the following two-agents ($N = \{1,2\}$) two-states $\Omega = \{1,2\}$ illustration. Restrict attention to social states where individual 1 earns 3 in the bad state 1 and earns 4 in the good state 2 while individual 2 earns 0.5 in the bad state and 2 in the good state (all other social states receiving zero probabilities). Consider specifically the socially risky situations p and q defined by:

$$\begin{aligned} p_{(1,3,2,2)} &= 1/2 \\ &= p_{(2,4,1,0.5)} \end{aligned}$$

and $p_x = 0$ for all other social states $x \in (\{1,2\} \times \mathbb{I})^n$ and:

$$\begin{aligned} q_{(1,3,1,1/2)} &= 1/2 \\ &= q_{(2,4,2,2)} \end{aligned}$$

and $q_x = 0$ for all other $x \in (\{1,2\} \times \mathbb{I})^n$. Notice that the two individuals face *ex ante* the same probability of being unemployed (and of getting their income contingent upon this status) in the two socially risky situations. Because of this, p and q are considered equivalent by either the SEHP or the SEPG criterion. What would be the verdict of the ex post need analysis ? This depends of course upon which of the two outcome arises in each of the two social lotteries. There are really only four possibilities here (all equally probable if the two socially risky situations are independent):

$$\text{outcome } (1,3,2,2) \text{ arises in } p \text{ and outcome } (1,3,1,1/2) \text{ arises in } q \tag{12}$$

$$\text{outcome } (1,3,2,2) \text{ arises in } p \text{ and outcome } (2,4,2,2) \text{ arises in } q \tag{13}$$

$$\text{outcome } (2,4,1,1/2) \text{ arises in } p \text{ and outcome } (1,3,1,1/2) \text{ arises in } q \tag{14}$$

$$\text{outcome } (2,4,1,1/2) \text{ arises in } p \text{ and outcome } (2,4,2,2) \text{ arises in } q \tag{15}$$

Applying the Bazen-Moyes-Jenkins-Lambert criterion to the comparisons of the two outcomes in each of the cases (12)-(15) yields a strict dominance of the *ex post* result of p over that of q in cases (12) and (14) and the reverse conclusion of a strict dominance of the *ex post* result of q over p in cases (13) and (15). Hence the comparisons of two particular *ex post* realizations of two socially risky situations do not provide much guidance for the ex ante comparison of these two situations.

We conclude this subsection by providing a normative foundation for each of these two criteria. More specifically, we show that each criterion coincides with the ranking of socially risky situations that commands unanimity over all anonymous, Paretian and VNM social rankings who assume that individual VNM preferences can be represented by expected utility functions in one the two classes defined above. The proofs of these two results, that are straightforward and use standard (discrete) integration by part arguments, have been relegated in the appendix.

The first theorem establishes the equivalence between normative dominance for the class $\mathbb{U}_1$ and SEHP dominance.

Theorem 1 *Let p and q be two socially risky situations in $\mathbb{L}$. Then $p \succsim_{\mathbb{U}_1} q$ if and only if $p \succsim_{SEHP} q$.*

The second theorem establishes the equivalence between normative dominance over the class $\mathbb{U}_2$ and sequential expected poverty gap dominance.

Theorem 2 *Let p and q be two socially risky situations in $\mathbb{L}$. Then $p \succsim_{\mathbb{U}_2} q$ if and only if $p \succsim_{SEPG} q$.*

3 Empirical illustration

We now illustrate how the implementable criteria proposed in the preceding section can generate interesting empirical conclusions. Our illustration is made using *sample* data. In order to derive from these sample data conclusions that are valid for the populations represented by the samples, we perform statistical inference based on the Union-Intersection (UI) method as initiated by Bishop, Formby, and Thistle (1989)[8]. The UI method asserts that one *does not reject* the hypothesis of dominance of a socially risky situation p over a socially risky situation q when none of the poverty differences that define the dominance criterion is significantly positive and at least one of them is significantly negative. The statistical inference methodology used in this paper is very close to the one used in

[8] See Howes (1994) for a critical appraisal of this inference methodology, that can be contrasted with another, more conservative, *Intersection-Union* (IU) one.

Gravel, Moyes, and Tarroux (2009) and we refer to that paper for further details. All comparisons presented herein are performed at the 95% confidence level.

Our illustration concerns the labour market in France and in the US.[9] Risks of job loss remain obviously a major concern for billions of individuals all over the world. This concern has fueled a literature that attempts at empirically measuring the phenomenon in various western countries. For the United States, authors like Farber (2004), have suggested that the average job insecurity in the US has increased mildly in the late nineties while others, like Gottschalk and Moffit (1999), have shown no evidence of an increase in the probability of loosing one's job. There has been also some papers that have examined the evolution of the average risk of involuntary job loss in France. For instance, Givord and Maurin (2004) suggests that the probability of involuntary job loss has increased since the 1980s. It has also been noticed by Postel-Vinay (2003) that the increase in the risk of loosing one's job has been larger for low-seniority workers than for high-seniority ones. These studies have focused on the average probability of being unemployed and have not derived meaningful normative conclusion out of their analysis. The criteria of the previous section are potentially useful for this purpose.

We illustrate this by comparing exposures to unemployment risks of single adult members of the workforce between US and France. We focus on single adults to avoid normatively challenging issues that concern multi-individual households. We use the French labour Force Survey (LFS) and the US Current Population Survey-March Supplement (CPS-MS) for both 2003 and 2004. The LFS contains 50,524 respondents (employees and unemployed), 6,953 of which being single individuals without children. In the US CPS, the number of respondents is 90,314 (employees and unemployed) and the number of those who are single without children is 7,523. In both data sets, the same individuals are observed in 2003 and 2004. The fact that some of them have experienced change in employment status between the two years enables us to assign to each individual a probability of being unemployed, an income if employed and a (substitution) income if unemployed.

Consider first the probability of unemployment. This probability means different things for different individuals. For an individual observed *unemployed* in 2003, this probability is the probability of remaining unemployed in 2004. For an individual observed *employed* in 2003, it is the probability of loosing his or her job between 2003 and 2004. We assign probabilities to individuals by grouping them into homogeneous groups with respect to observable characteristics and by assigning to each individual of the group the same probability of unemployment. We formed 38 groups of individuals who were employed in 2003 (according to their level of education, activity sector, age and the fact that they work in the private or the public sector) and 10 groups of unemployed individ-

[9] Another empirical use of the criteria developed in this paper to understand the evolution of the exposures of Indian citizens to risks of death is provided in Gravel, Mukhopadhyay, and Tarroux (2008).

uals (defined on the basis of education, unemployment seniority and gender). While the French LFS distinguishes between voluntary and involuntary unemployment, the CPS does not. Hence, we adjusted our estimated risks in the US by using the Displaced Workers Survey (DWS). The DWS is conducted in January only on the same sample of individuals used in the CPS. It asks workers whether or not they were involuntary displaced from a job at any time in the preceding three-year period. We use the DWS to estimate the fraction of unemployed individuals who have been involuntary put into that situation.

As for the activity income, we have assigned to every individual employed in 2003 the observed income of this individual provided by the data. For an individual observed unemployed in 2003, we assign to him or her the monthly labour income he or she would have earned had he or she been employed. In order to assign this income, we estimate a wage equation on the sample of employed individuals. We of course account for the possible selection bias that could arise from the fact that we assign to unemployed individuals a wage that has been estimated on a sample of employed households by using Heckman (1979)'s methodology. The independent variables used in the wage equation are seniority (dummy), occupations (6 dummies), industries (5 dummies), city size (10 dummies for France and 8 for United States), education level (6 dummies), age and age squared. The independent variables used in the selection equation of the Heckman procedure - the fact of being employed - are, in addition to those, the number of children between 6 and 18, the number of children below 6 and dummy variables of the marital status (divorced and widows). We have performed the estimation separately for the samples of female and male singles. The activity income obtained is then transformed for all individuals into disposable income by subtracting income taxes (net of possible income tax credit) and by adding welfare payments, if any.

Finally, we assign to each individual a replacement income received in case of unemployment on the basis of the legislation in the two countries. This substitution income is principally made of unemployment benefits and/or social welfare payments. Unemployment benefits are function of the past activity income and the intensity of work (full/part time). Unemployment benefits are more generous in France (where they can last for one year) than in US (where they do not go beyond 26 weeks). In France the only considered welfare payment is the *Revenu Minimum d'Insertion* (RMI) (about $400 US a month) that works like a minimal income. Since much welfare payments in the US are given to family with at least one child, we ignore these benefits in this study devoted to single adults. Other welfare payments like Housing benefits ("*Allocation Personnalisée au Logement*" and "*Allocation Logement*" in France, and Low-Rent Public Housing and Housing Choice Vouchers in the US) are also ignored because we do not have information on housing prices.

Summary statistics on probability estimates and average activity and replacement incomes are provided in table 2. All pecuniary figures are in US dollars corrected for purchasing power parity.

As can be seen, the probability of being (becoming or remaining) employed is both higher and more equally distributed amongst single workers in the US than in France. Notice that this appears to be true only for those individuals who were unemployed in 2003, as unemployment inertia is stronger in France than in the US. Among the employed individuals, there does not seem to be much difference between the probability of keeping one's job for one year in France (95.99 %) and in the US (95.51 %). This however seems to be specific to the population of single adults without children. The other estimations that we have done for the two populations suggest that, if we include the other members of the workforce, the probability of keeping one's job is also significantly higher in US than in France. Moreover France seems also to be more "unequal" than in the US in terms of the way it distributes the probability of keeping one's job across its single adults. The gap in average probability of good state is larger between women and men and between "old" (above thirty) and "young" workers in France than in the United States. We can also note that women seem to face, in both countries, a lower probability of being unemployed. This can be explained in part by the fact that there is a larger proportion of women working in the less risky public sector.

	France	United States
Probability of employment (%)	88.67	93.84
	(19.88)	(12.74)
Female	89.66	94.64
	(18.68)	(12.17)
Male	87.90	93.13
	(20.74)	(13.18)
<30 old	87.08	93.66
	(19.32)	(12.33)
>30 old	89.4	93.90
	(20.10)	(12.87)
Probability of remaining employed	95.99	95.51
	(3.97)	(9.45)
Probability of becoming employed	39.93	69.68
	(13.81)	(24.33)
Monthly income (PPP $)		
Mean income in employment	1,284	2,508
	(685.28)	(2428)
Mean replacement income	885	856
	(564.05)	(362.81)

Table 2: summary statistics, France USA

As a benchmark for the analysis conducted in this section, we show in Figure 3a and 3b the conventional one-dimensional poverty gap curves for men and women in the US and in France. These curves are drawn by considering, on the horizontal axis, the individual monthly income (net of taxes and transfers) as observed in 2003 *without* taking into account the employment status of the individual. To that extent, and referring back to the discussion of the previous section, these curves are drawn *ex-post* after the assignment of individuals to their employment status. The verification that one of these two curves lies everywhere below the other would correspond to the second step of Jenkins and Lambert (1993) or Bazen and Moyes (2003) procedure (the first stage being the drawing of these two curves on the sole population of unemployed). As can be seen, except for very small poverty lines (see figure 3a), the amount of income that is required for eliminating poverty tends to be much larger in France than in the US. Non surprisingly, the within-country ranking of the men and women subsamples (favorable to men) is the same in the two countries. differ somewhat in the two countries. In France, and with the exception of relatively high income thresholds, the fraction of women who are poorer than any threshold is larger than the corresponding fraction for men, even for very low poverty lines. In the US, this (headcount) poverty dominance of men over women is not observed for very low incomes. Hence, in the US, men tend to be more affected by severe poverty than women while the converse is true in France.

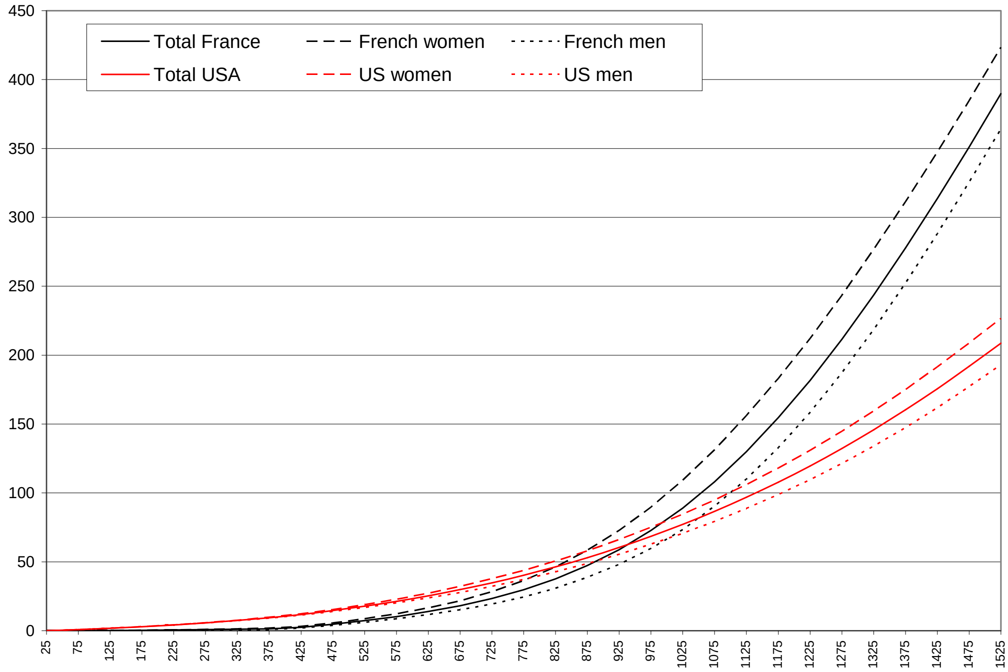

Figure 3a: Conventional expected poverty gap curves for low income individuals.

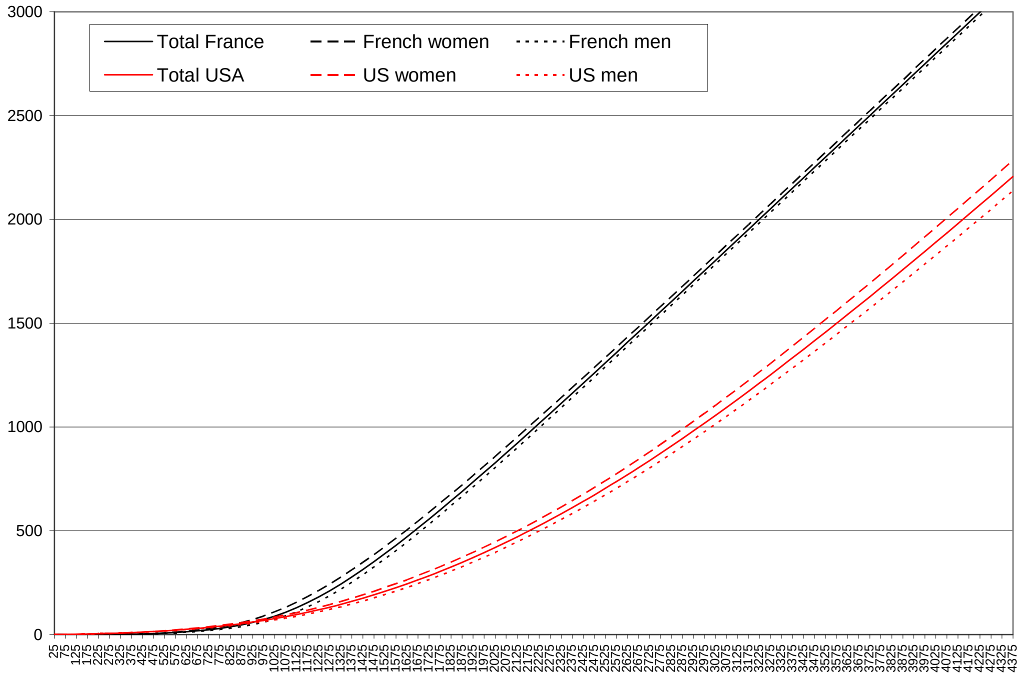

Figure 3b: Conventional poverty gap curves for the whole population.

We now compare the two countries using our *ex ante* SEHP and SEPG criteria. Doing this requires us to order the two states of nature that define employment status. There are really only three possibilities here:

1) unemployment is intrinsically worse than employment,

2) unemployment is intrinsically better than employment and,

3) unemployment risks have only pecuniary consequences so that none of the two states is intrinsically better than the other.

The first possibility corresponds to a widespread view, supported by some empirical evidence (see e.g. Clark and Oswald (1994)), that unemployment has important adverse non-pecuniary consequences (social stigma, loss of self-esteem, etc.) that outweigh the possible non-pecuniary benefit associated to the extra leisure time that it provides. The second possibility reflects the converse belief that the non pecuniary benefit of leisure time dominates the non-pecuniary cost of unemployment. The third possibility assumes that non-pecuniary negative and positive consequences of unemployment either cancel out each other or that their net effect is sufficiently small as compared to the pecuniary consequences that they can be neglected for all practical purposes. As we do not want to take any firm stance as to which of the three possibilities is the more likely, we apply the criteria to each of them in turn.

Figures 4a, 4b and 4c provide expected poverty gap curves in the unemployment state, the employment state and in either state respectively for the total adult population of the two countries as well as the male and female subsamples. Recall from the definition of SEPG dominance that non-crossing of two curves in figures 4a and 4c is required if unemployment is considered to be a worse state than employment (first possibility). If the second possibility is considered, then non-crossing of two curves in figures 4b and 4c is required. Finally, if one assumes that unemployment risk has only pecuniary consequences, then the criterion that applies in that case only requires non-crossing of the two curves in figure 4c. It can be noticed that the *ex ante* poverty curves shown in figure 4c (used in the second stage of our SEPG criterion) are different from the *ex post* ones shown in figure 3b, which would be used in the second stage of an analysis à la Bazen and Moyes (2003) or Jenkins and Lambert (1993). Particularly important is the difference concerning the within-country comparisons of these curves for the male and female populations.

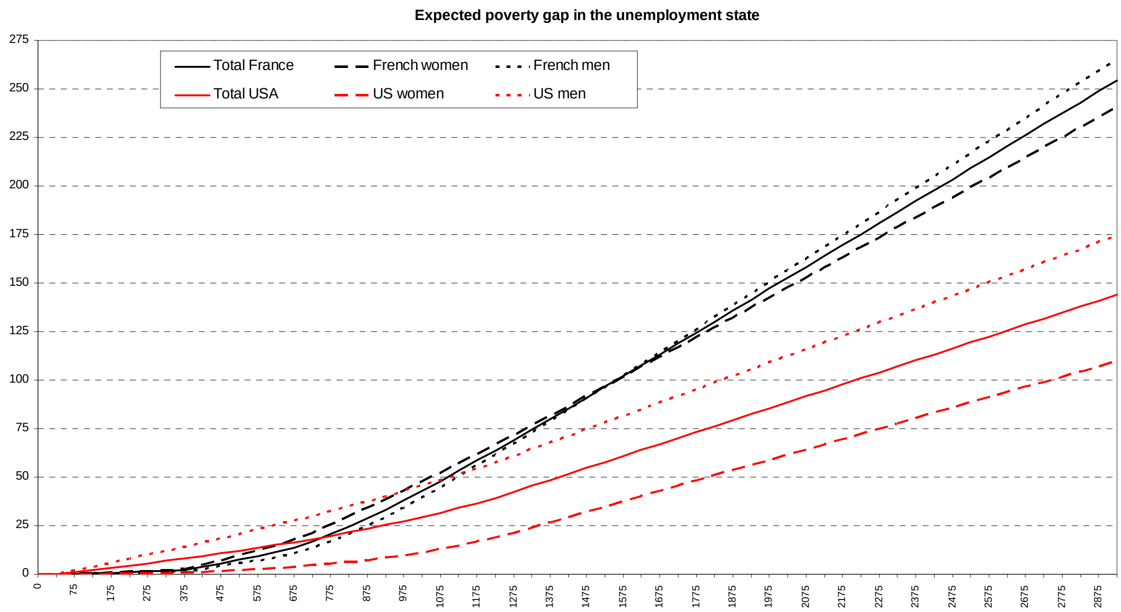

Figure 4a

Since a crossing of any two curves on figure 4c is sufficient to block a dominance verdict for any of the three orderings, it is clear that, irrespective of the population considered - male, female or the whole - there is *no* dominance between France and the US. As appears clearly on figure 4c, the French curves lie below their US counterparts for very low levels of income (less than $900 a month) while they lie above for the rest of the distribution. This crossing of the curves at $900 happens of course because of the protection provided to extremely poor workers in France by the RMI, a protection that is not present in the US. Of course, only statistical testing can determines if differences and crossing between curves are significant.

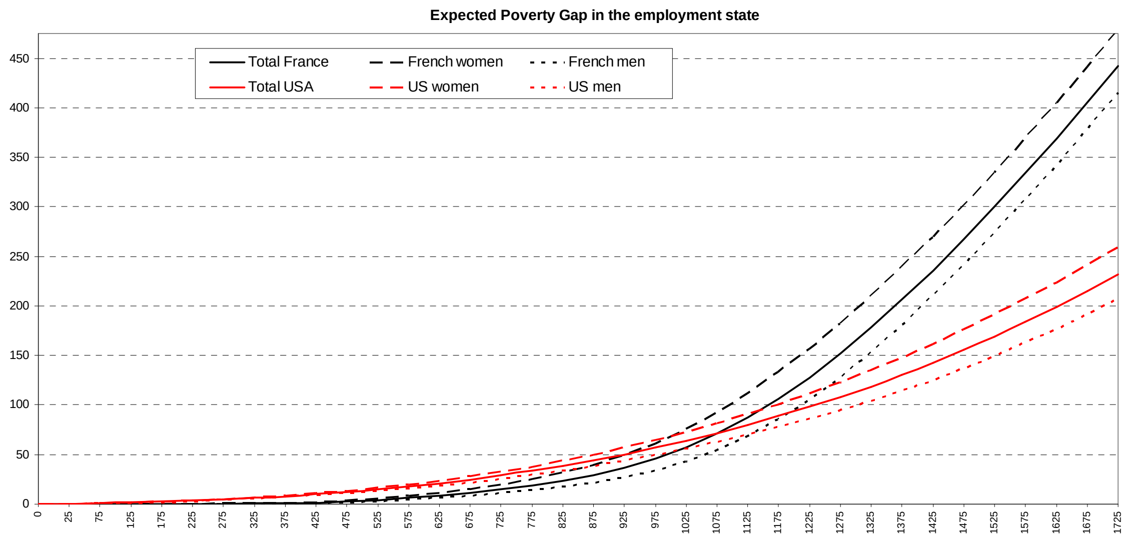

Figure 4b

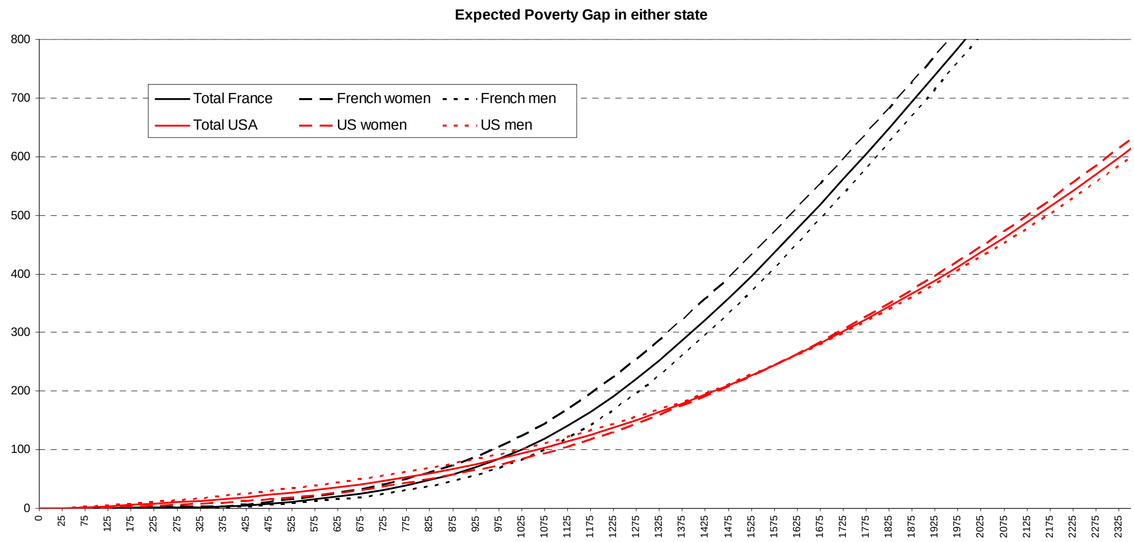

Figure 4c

Interesting also, especially in view of what was suggested by figures 3a and 3b, are the comparisons of the male and female curves within each country. In the US, expected poverty gap irrespective of the state is higher for women than for men at the upper tail of the distribution but lower at the bottom part of it. On the other hand, if one concentrates on unemployed individuals, expected poverty gap is everywhere lower for women than for men. The situation is different in France where expected poverty gap irrespective of the state is lower for men than for women both in the unemployed state and irrespective of the states. Hence, while men seem to be robustly more protected against unemployment risk than women in France, no such dominance arises in the US. In this sense, it can be said that the men-women gap in terms of protection against unemployment risks is lower in the US than in France.

Table 3 shows the results of the comparisons, on the basis of the expected poverty gap criteria, of the above populations for all three orderings of the states based on statistical inference, rather than visual inspection of graphs. The table also shows the results of the comparisons, within each country, of young and old segments

Comparison SEPG	**Ranking**	Min t	Max t	degree of freedom	critical t
FRANCE-US					
unemployment is bad	**?**	-18.07	63.32	$(396,\infty)$	3.95
employment is bad	?	-18.07	63.32	$(403,\infty)$	3.96
state independent	?	-18.07	63.32	$(201,\infty)$	3.66
FRANCE					
Fem.-Male, unemployment is bad	**?**	-3.71	9.57	$(158,\infty)$	3.59
Fem-Male, employment is bad	**Male**	-1.07	10.77	$(179,\infty)$	3.63
Fem-Male, state independent	**Male**	2.2	9.57	$(89,\infty)$	3.44
Young-old, unemployment is bad	**OLD**	4.67	12.38	$(157,\infty)$	3.59
Young-old, employment is bad	**?**	-4.43	12.38	$(179,\infty)$	3.63
Young-old, state independent	**OLD**	6.37	12.38	$(89,\infty)$	3.44
US					
Fem-Male, unemployment is bad	?	-17.43	5.63	$(203,\infty)$	3.66
Fem-Male, employment is bad	?	-10.83	9.2	$(361,\infty)$	3.9
Fem-Male, state independent	?	-10.83	5.63	$(180,\infty)$	3.63
Old-Young, unemployment is bad	**OLD**	-1.96	14.65	$(203,\infty)$	3.66
Old-Young, employment is bad	**OLD**	3.36	14.65	$(361,\infty)$	3.9
Old-Young, state independent	**OLD**	1.7	14.65	$(180,\infty)$	3.63

Table 3: SEPG comparisons

of the single adults sample (irrespective of gender), with 30 years old as the cut-off age.

Table 3 thus reinforces the impression, provided by figures 4a-c, on the France-US differences with respect to the way they expose their adult males and females to unemployment risks. Except when unemployment is assumed to be the worst state, the exposure of French male adults to unemployment risks is better than that of female. Moreover, the failure to achieve dominance of male over female in the case where unemployment is the worse state only comes from the fact that average unemployment is lower in France for females than for males (remember that condition (9) must hold in order to have SEPG dominance). Yet, if one is willing to assume that, for very high income, there is no utility difference between being unemployed and being employed - a reasonable assumption for unemployment - then one would conclude that French men are better protected than French women against unemployment risks. Table 3 also reveals that, as can be expected, the old segment of the adult population is, in both countries, better protected against unemployment risks than the young segment. As it turns out, this result holds even if one uses the more robust SEHP criterion. Notice that, in the case of France, the dominance of old adults over young ones does not hold if one assumes that employment is a worse state than unemployment at a given income level. The reason for this dominance failure comes, here again, from the failure to satisfy condition (9) (the probability of being employed is significantly higher in France among the old than among the young).

4 Conclusion

This paper characterizes robust criteria for comparing socially risky situations from a normative point of view. The criteria characterized are SEHP and SEPG dominance. Each of these criteria is shown to coincide with the ranking that commands unanimous agreement among all VNM social rankings that are anonymous and Pareto inclusive with respect to a specific wide class of individualistic VNM preferences. While the sequential expected poverty dominance criteria that we obtained are evocative of two-dimensional dominance criteria *à la* Atkinson and Bourguignon (1987), they use the *ex ante* probabilities faced by individuals to weight the poverty assigned to each of them rather than the *ex post* frequency of these individuals in the population. As illustrated in our empirical application on the distribution of risks of involuntary unemployment in France and the US, the dominance criteria are easy to use, and are capable of producing interesting conclusions. Among other things, it happens that a statistically significant dominance of men over women in terms of exposure to risks of involuntary unemployment as per our criteria is not observed in the US while it is observed in France. This suggests that the later country can be seen as "more unfair" than the former in terms of its allocation of risks of involuntary unemployment between men and women.

There are various directions in which the analysis of this paper could be extended. First, it would be interesting to apply the criteria in a dynamic perspective so as to see whether the folk impression of a growing exposure of workers to unemployment risks within countries is indeed plausible. Another interesting extension of the analysis would be to construct and implement summary indices of risk exposure that would enable a complete ranking of risky situations in a way that is compatible with their incomplete ranking provided by the dominance criteria. We believe also the criteria to be potentially useful to study the theoretical design of normatively appealing systems of unemployment insurance, in a similar way than the Lorenz criterion is used, in the Jakobsson

(1976) tradition, to analyze the progressivity of a tax schedule in the context of income distributions.

Last, but not least, it would be interesting to examine alternative dominance criteria that could be based on different assumptions on the properties of the individual state-dependent preferences. Two properties that are somewhat disputable in the criteria considered herein are those that assert that the marginal utility of income decreases with the state (healthy people get less extra pleasure from money than less healthy ones) and that the degree of absolute risk aversion (as measured by the Arrow-Pratt coefficient) is also decreasing with the state. Our preliminary investigations suggest that it is possible to derive empirical criteria that make the converse assumptions on the impact of the state of the world on both the marginal utility of income and the decrease in the marginal utility of income. Pursuing these investigations is a well-worth objective of future research.

A Proofs of theorems 1-2

A.1 Proof of theorem 1

For the first implication, assume that $p \succsim_{\mathbb{U}_1} q$. Then, the inequality:

$$\sum_{x\in\mathbb{X}} p_x \sum_{i\in N} U(j_i^x, y_i^x) \geq \sum_{x\in\mathbb{X}} q_x \sum_{i\in N} U(j_i^x, y_i^x) \tag{16}$$

holds for every function $U : \Omega \times \mathbb{I} \to \mathbb{R}$ in $\mathbb{U}_1$. Consider, for any $k \in \{1, ..., l\}$ and $t \in \mathbb{I}$, the function $V^{kt} : \Omega \times \mathbb{I} \to \mathbb{R}$ defined, for any $j \in \Omega$ and $y \in \mathbb{I}$ by:

$$\begin{aligned} V^{kt}(j,y) &= -1 \text{ if } y \leq t \text{ and } j \leq k \\ &= 0 \text{ otherwise} \end{aligned}$$

It can be checked that the function V^{kt} so defined belongs to $\mathbb{U}_1$ for any $k \in \{1, ..., l\}$ and $t \in \mathbb{I}$. Hence, inequality (16) holds for the functions V^{kt} so that we have:

$$\begin{aligned} \sum_{x\in\mathbb{X}} p_x \sum_{i\in N} V^{kt}(j_i^x, y_i^x) &\geq \sum_{x\in\mathbb{X}} q_x \sum_{i\in N} V^{kt}(j_i^x, y_i^x) \\ &\Leftrightarrow \\ \sum_{i\in N} \sum_{\{x\in\mathbb{X}: j_i^x \leq k \;\&\; y_i^x \leq t\}} -p_x &\geq \sum_{i\in N} \sum_{\{x\in\mathbb{X}: j_i^x \leq k \;\&\; y_i^x \leq t\}} -q_x \\ &\Leftrightarrow \\ \sum_{i\in N} \sum_{\{x\in\mathbb{X}: j_i^x \leq k \;\&\; y_i^x \leq t\}} p_x &\leq \sum_{i\in N} \sum_{\{x\in\mathbb{X}: j_i^x \leq k \;\&\; y_i^x \leq t\}} q_x \end{aligned}$$

as required by (6).

For the other implication, consider a subdivision of the interval $[0,1]$ into r sub-intervals $[\rho_h, \rho_{h+1}]$ for $h = 0, ..., r-1$ such that:

$$\begin{aligned}\rho_0 &= 0 \\ \rho_m &= 1\end{aligned}$$

and, for all $i \in N$, $j \in \Omega$ and $y \in \mathbb{I}$, there are h and $h' \in \{0, 1, ..., r\}$ such that $p(i,j,y) = \rho_h$ and $q(i,j,y) = \rho_{h'}$. We this notation, we can write (16) as:

$$\sum_{h=1}^{r} \sum_{y=1}^{m} \sum_{j=1}^{l} \Delta f_j(\rho_h, y) \rho_h U(j,y) \geq 0 \tag{17}$$

where, for $h = 1, ..., r$, $j = 1, ...l$ and $y = 1, ..., m$,

$$\begin{aligned}\Delta f_j(\rho_h, y) &= \#\{i \in N : p(i,j,y) = \rho_h \} \\ -\#\{i &\in N : q(i,j,y) = \rho_h\}\end{aligned} \tag{18}$$

(we of course allow for the possibility that the cardinality of either of the two sets that enters in (18) be zero). We now proceed by decomposing the left hand side of (17) using Abel identity (see for instance (Fishburn and Vickson (1978); eq 2.49)). Doing first the decomposition with respect to the y-indexed summation operator yields:

$$\sum_{h=1}^{r} \sum_{j=1}^{l} [\sum_{y=1}^{m} \Delta f_j(\rho_h, y) \rho_h U(j,m) - \sum_{t=1}^{m-1} \sum_{y=1}^{t} \Delta f_j(\rho_h, y) \rho_h (U(j,t+1) - U(j,t))] \geq 0 \tag{19}$$

Decomposing (19) using Abel identity applied this time to the j-indexed sum operator yields:

$$\sum_{h=1}^{r} [\sum_{j=1}^{l} \sum_{y=1}^{m} \Delta f_j(\rho_h, y) \rho_h U(l,m) - \sum_{k=1}^{l-1} (\sum_{j=1}^{k} \sum_{y=1}^{m} \Delta f_j(\rho_h, y) \rho_h)(U(k+1,m) - U(k,m))$$

$$- \sum_{j=1}^{l} \sum_{t=1}^{m-1} \sum_{y=1}^{t} \Delta f_j(\rho_h, y) \rho_h (U(l,t+1) - U(l,t))$$

$$+ \sum_{k=1}^{l-1} \sum_{t=1}^{m-1} \sum_{j=1}^{k} \sum_{y=1}^{t} \Delta f_j(\rho_h, y) \rho_h (U(k+1,t+1) - U(k+1,t) - U(k,t+1) + U(k,t))] \geq 0 \tag{20}$$

Now, using (18), one can see that that, for every $t \in \mathbb{I}$ and $k \in \Omega$:

$$\sum_{h=1}^{r} \sum_{j=1}^{k} \sum_{y=1}^{t} \Delta f_j(\rho_h, y) \rho_h = \sum_{i \in N} \sum_{j \leq k} \sum_{y \leq t} p(i,j,y) - \sum_{i \in N} \sum_{j \leq k} \sum_{y \leq t} q(i,j,y) \tag{21}$$

Combining this with the fact that:

$$\sum_{h=1}^{r}\sum_{j=1}^{l}\sum_{y=1}^{m}\Delta f_j(\rho_h,y)\rho_h=\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in\mathbb{I}}p(i,j,y)-\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in\mathbb{I}}q(i,j,y)=n-n=0$$

we can write (20) as:

$$\begin{gathered}-\sum_{g=1}^{l-1}(\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}(p(i,j,y)-q(i,j,y)))(U(k+1,m)-U(k,m))\\ -\sum_{k=1}^{m-1}(\sum_{i\in N}\sum_{j\leq l}\sum_{y\leq t}(p(i,j,y)-q(i,j,y)))(U(l,t+1)-U(l,t))\\ +\sum_{g=1}^{l-1}\sum_{k=1}^{m-1}(\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}(p(i,j,y)-q(i,j,y)))(U(k+1,t+1)-U(k+1,t)-U(k,t+1)+U(k,t))\geq 0\end{gathered}\tag{22}$$

As can be seen, having:

$$\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}p(i,j,y)\leq\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}q(i,j,y)$$

for all $j\in\Omega$ and all $k\in\mathbb{I}$ is sufficient for inequality (22) to hold for all state-dependent utility functions U_j in $\mathbb{U}_1$.

A.2 Proof of theorem 2.

Assume first that $p\succsim_{\mathbb{U}_2}q$ and, accordingly, that inequality (16) holds for all utility functions $U:\Omega\times\mathbb{I}\to\mathbb{R}$ in $\mathbb{U}_2$. Consider, for any $k\in\Omega$ and $t\in\mathbb{I}$, the function $\widetilde{V}^{kt}:\Omega\times\mathbb{I}\to\mathbb{R}$ defined, for $j\in\Omega$ and $y\in\mathbb{I}$, by:

$$\begin{aligned}\widetilde{V}^{kt}(j,y) &= \min(y-t,0)\text{ if }j\leq k\\ &= 0\text{ otherwise}\end{aligned}$$

For a given $j\in\Omega$, the function $\widetilde{V}^{kt}$ is the "angle" function used in the classical proof of the Hardy-Littlewood-Polya theorem made by Berge (1959). The reader can verify that the functions $\widetilde{V}^{kt}$ belong to $\mathbb{U}_2$. For this reason the inequality:

$$\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in\mathbb{I}}p(i,j,y)\widetilde{V}^{kt}(j,y)\geq\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in\mathbb{I}}q(i,j,y)\widetilde{V}^{kt}(j,y)\tag{23}$$

holds for every k and t. Using the definition of the functions $\widetilde{V}^{kt}$, inequality (23) writes:

$$\begin{array}{rcl}\sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}p(i,j,y)\min(y-t,0) & \geq & \sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}q(i,j,y)\min(y-t,0) \\ & \Leftrightarrow & \\ \sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}p(i,j,y)\max(t-y,0) & \leq & \sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}q(i,j,y)\max(t-y,0)\end{array}$$

as required by condition (8) of SEPG dominance. To obtain condition (9) of SEPG dominance, we consider, for every $k\in\Omega$, the functions $V^k\colon\ \Omega\times\mathbb{I}\to\mathbb{R}$ defined, for $j\in\Omega$ and $y\in\mathbb{I}$, by:

$$\begin{array}{rcl}V^k(j,y) & = & -1 \text{ if } j\leq k \\ & = & 0 \text{ otherwise}\end{array}$$

These k-indexed functions clearly satisfy $V^k(j+1,y)\geq V^k(j,y)$ for every $y\in\mathbb{I}$ and $j=1,...,l-1$ and are all (trivially) increasing with respect to income for every j. It can be checked that these functions satisfy (very often trivially) the conditions imposed on the functions in $\mathbb{U}_2$. Hence, inequality (16) holds for any such functions V^k so that we have, for all $k\in\Omega$:

$$\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in\mathbb{I}}p(i,j,y)V^k(j,y)\geq\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in\mathbb{I}}q(i,j,y)V^k(j,y)$$

or

$$\begin{array}{rcl}\sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}-p(i,j,y) & \geq & \sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}-q(i,j,y) \\ & \Leftrightarrow & \\ \sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}p(i,j,y) & \leq & \sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}q(i,j,y)\end{array}$$

as required by condition (9).

For the other implication, it proceeds just as in the proof of theorem 1 by writing inequality (16) in the form of (17) and by doing the Abel decomposition of (17) until one reaches condition (20). If one then goes one step further and Abel decomposes each term of (20) with respect to the inner (y-indexed) term, one obtains:

$$\begin{array}{l}\sum_{h=1}^{r}[-\sum_{k=1}^{l-1}\sum_{j=1}^{k}\sum_{y=1}^{m}\Delta f_j(\rho_h,y)\rho_h(U(k+1,m)-U(k,m)) \\ -\sum_{t=1}^{m-1}\sum_{y=1}^{t}\sum_{j=1}^{l}\Delta f_j(\rho_h,y)\rho_h(U(l,t+1)-U(l,t)) \\ +\sum_{v=1}^{m-2}\sum_{t=1}^{v}\sum_{y=1}^{t}\sum_{j=1}^{l}\Delta f_j(\rho_h,y)\rho_h(U(l,v+2)-2U(l,v+1)+U(l,v))\end{array}$$

$$+\sum_{k=1}^{l-1}\sum_{t=1}^{m-1}\sum_{j=1}^{k}\sum_{y=1}^{t}\Delta f_j(\rho_h,y)\rho_h(U(k+1,t+1)-U(k+1,t)-U(k,t+1)+U(k,t))$$

$$-\sum_{k=1}^{l-1}\sum_{v=1}^{m-2}(\sum_{t=1}^{v}\sum_{j=1}^{k}\sum_{y=1}^{t}\Delta f_j(\rho_h,y)\rho_h\Delta^{U^2}(k,v))]\geq 0 \tag{24}$$

where:

$$\begin{aligned}\Delta^{U^2}(k,v) &= U(k+1,v+2)-2U(k+1,v+1)+U(k+1,v)\\ &\quad -[U(k,v+2)-2U(k,v+1)+U(k,v)]\end{aligned}$$

Noticing that:

$$\begin{aligned}\sum_{h=1}^{m}\sum_{j=1}^{k}\sum_{t=1}^{v}\sum_{y=1}^{t}\Delta f_j(\rho_h,y)\rho_h &= \sum_{t=1}^{v}[\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}[p(i,j,y)-q(i,j,y)]\\ &= \sum_{t=1}^{v}(\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}[p(i,j,y)-q(i,j,y)])(t-y)\\ &= \sum_{t=1}^{v}(\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}[p(i,j,y)-q(i,j,y)])P(t,y)\end{aligned}$$

and remembering equation (21) in the proof of theorem 1, one can write (24) as:

$$\begin{aligned}&-\sum_{k=1}^{l-1}(\sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}(p(i,j,y)-q(i,j,y))(U(k+1,m)-U(k,m))\\ &-(\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\leq m-1}(p(i,j,y)-q(i,j,y))P(m-1,y)(U(l,m)-U(l,m-1))\\ &+\sum_{t=1}^{m-2}(\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\leq t}(p(i,j,y)-q(i,j,y)P(t,y))(U(l,t+2)-2U(l,t+1)+U(l,t))\end{aligned}$$

$$+\sum_{k=1}^{l-1}(\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}(p(i,j,y)-q(i,j,y))P(t,y))(U(k+1,t+1)-U(k+1,t)-U(k,t+1)+U(k,t))$$

$$-\sum_{k=1}^{l-1}\sum_{v=1}^{m-2}(\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t}(p(i,j,y)-q(i,j,y))P(t,y)\Delta^{U^2}_{kv})]\geq 0 \tag{25}$$

For any combination of functions U_j (for $j\in\Omega$) belonging to $\mathbb{U}_2$, it is sufficient for (25) to hold to have, for all $k\in\Omega$:

$$\sum_{i\in N}\sum_{j\leq k}\sum_{y\in\mathbb{I}}(p(i,j,y)-q(i,j,y))\leq 0,$$

and:

$$\sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t} p(i,j,y)P(t,y) - \sum_{i\in N}\sum_{j\leq k}\sum_{y\leq t} q(i,j,y)P(t,y) \leq 0$$

for all $t \in \mathbb{I}$, which is the definition of SEPG dominance.

References

Atkinson, A. B., and F. Bourguignon (1982): "The Comparison of Multidimensioned Distribution of Economic Status," *Review of Economic Studies*, 49, 183–201.

——— (1987): "Income Distributions and Differences in Needs," in *Arrow and the Foundation of the Theory of Economic Policy*, ed. by G. R. Feiwel. Macmillan, London.

Bazen, S., and P. Moyes (2003): "Comparisons of income distributions accross heterogenous populations," *Research on Economic Inequality*, 9, 85–115.

Berge, C. (1959): *Espaces topologiques et fonctions multivoques.* Dunod, Paris.

Bishop, C. M., J. P. Formby, and P. D. Thistle (1989): "Statistical Inference, Income Distributions and Social Welfare," in *Research on Economic Inequality*, ed. by D. J. Slotje. JAI Press.

Clark, A. E., and A. J. Oswald (1994): "Happiness and Unemployment," *The Economic Journal*, 104, 648–659.

Dutta, I., J. Foster, and A. Mishra (2011): "On Measuring Vulnerability to Poverty," *Social Choice and Welfare*, 37, 743–761.

Farber, H. S. (2004): "Job loss in the United States, 1981-2001," *Research in Labor Economics*, 23, 69–117.

Fishburn, P. C., and R. G. Vickson (1978): "Theoretical Foundations of Stochastic Dominance," in *Stochastic Dominance*, ed. by G. A. Withmore, and M. C. Findlay. Lexington Books.

Fleurbaey, M. (2010): "Assessing Risky Social Situations," *Journal of Political Economy*, 118, 649–680.

Givord, P., and E. Maurin (2004): "Changes in Job Security and their Cause: an Empirical Analysis for France, 1982-2002," *European Economic Review*, 48, 595–615.

Gottschalk, P., and R. Moffit (1999): "Changes in Job Instability and Insecurity using Monthly Survey Data," *Journal of Labor Economics*, 17, S91–S126.

Gravel, N., P. Moyes, and B. Tarroux (2009): "Robust International Comparisons of Distributions of Disposable Income and Access to Regional Public Goods," *Economica*, 76, 432–461.

Gravel, N., and A. Mukhopadhyay (2010): "Is India better off now than fifteen years ago ? A robust multidimensional answer," *Journal of Economic Inequality*, 8, 173–195.

Gravel, N., A. Mukhopadhyay, and B. Tarroux (2008): "A Robust Normative Evaluation of India's Performance in Allocating Risks of Death," *Indian Growth and Development Review*, 1, 95–111.

Harsanyi, J. (1955): "Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility," *Journal of Political Economy*, 63, 309–321.

Heckman, J. J. (1979): "Sample Selection Bias as a Specification Error," *Econometrica*, 47, 153–161.

Howes, S. (1994): "Testing for Dominance: Inferring Population Rankings from Sample Data," unpublished paper, policy research department, World Bank.

Jakobsson, U. (1976): "On the Measurement of the Degree of Progression," *Journal of Public Economics*, 8, 161–168.

Jenkins, S. P., and P. J. Lambert (1993): "Ranking Income Distributions when Needs Differ," *Review of Income and Wealth*, 39, 337–356.

Kolm, S. C. (1977): "Multidimensional Egalitarianisms," *Quarterly Journal of Economics*, 91, 1–13.

Postel-Vinay, F. (2003): "Evolution du Risque de Perte d'Emploi: Changements Structurels ou Changements Institutionnels ?," *Economie et Statistiques*, 366, 24–29.

Viscusi, W. K. (2007): "Regulation of Health, Safety and Environmental Risks," in *Handbook of Law and Economics*, ed. by A. M. P. . S. Shavell. Elsevier.

Viscusi, W. K., and W. M. Evans (1990): "Utility Functions that Depend on Health Status: Estimates and Economic Implications," *American Economic Review*, 80, 353–374.

Weymark, J. A. (1991): "A Reconsideration of the Harsanyi-Sen Debate on Utilitarianism," in *Interpersonal Comparisons of Well-Being*, ed. by J. Elster, and J. Roemer, pp. 255–320. Cambridge University Press, Cambridge, U. K.